# FIXED-POINT THEOREMS IN GENERALIZED METRIC SPACES UNDER DIFFERENT CONTRACTION CONDITIONS

## SHORT SYNOPSIS

*Submitted in partial fulfillment of the requirement of the degree of*

## DOCTOR OF PHILOSOPHY

*by*

**Nikita Mittal**

Registration No.- MRUMADN012K220054

Under the supervision of

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**MANAV RACHNA UNIVERSITY**

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Sector-43, Aravali Hills, Faridabad

Date: 13/06/2024

# DECLARATION BY THE SCHOLAR

I hereby declare that the short synopsis entitled "Fixed-Point Theorems in Generalized Metric Spaces Under Different Contraction Conditions" being submitted in partial fulfillment of the requirement for the degree of Doctor of Philosophy in the Department of Sciences under the school of Sciences of Manav Rachna University, Faridabad during the academic year 2023-2024.

It is the work that is expected to be carried out under the supervision of Dr. Aparna Vyas and co-supervision of Dr. Z.K. Ansari.

Further, it is declared that the short synopsis is within the plagiarism limit and a copy of the same is attached as per the University norms.

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## CERTIFICATE BY SUPERVISOR(S)

This is to certify that, the short synopsis entitled "Fixed-Point Theorems in Generalized Metric Spaces Under Different Contraction Conditions" is an original piece of work done by research scholar Nikita Mittal under the supervision of Dr. Aparna Vyas and co-supervision of Dr. Z.K. Ansari for the degree of Doctor of Philosophy in the Department of Sciences under the School of Sciences of Manav Rachna University, Faridabad during the academic year 2023-2024.

Further, it is certified that the short synopsis is within the plagiarism limit and copy of the same is attached as per the University norms.


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# ABSTRACT

This short synopsis gives an overview of some fixed-point results in generalized metric spaces under different contraction conditions. Our current research endeavors aim to expand the Banach contraction principle in many ways for example by easing the conditions or generalizing the underlying metric space and / or relaxing the contraction types. The short synopsis comprises the following **seven sections**.

**Section 1** gives a brief introduction of the history and development of fixed-point theory and is divided into **two subsections**. The definitions of the several generalized metric spaces that have been discussed and introduced in the literature are provided with examples in **Section 1.1**. **Section 1.2** discusses basic concepts and topological notions used in generalized metric spaces.

**Section 2** titled Literature Review summarizes the main work done by various authors related to my present research area.

**Section 3** gives the description of broad area.

**Section 4** addresses various objectives and goals of the research.

**Section 5's** objective is to analyze the research methods.

**Section 6 and 7** discusses expected outcomes of the research and list of references respectively.



**Key words**: Fixed point, Contraction map, Dislocated metric space, Dislocated quasi metric space, b-metric space, Extended b-metric space, Cauchy sequence.

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## 1. Introduction

One of the most well-known and progressive areas of nonlinear functional analysis study is the theory of fixed points. It deals with existence and uniqueness of fixed point $(S(\mu) = \mu)$, that is, a point that does not change under a given map. A mapping may or may not have any fixed points; if it does, there may be one single fixed point or several. Many types of fixed points such as coincidence point, common fixed point, coupled fixed point and many more, may be found in literature. Fixed point theorems are those that address the presence and characteristics of fixed points. In this field, the first fixed point theorem was established in 1912 by Brouwer [9], which asserts that "In $R^n$, any continuous function from a closed unit ball $C = \{\mu: \|\mu\| \leq 1\}$ to itself has a fixed point". It is a non-constructive result. After Brouwer [9], one of the most discussed results was proved by S. Banach [4] in 1922 for a contraction mapping in a complete metric space, well known as 'Banach contraction principle'. For a metric space $(\Im, \mathfrak{d})$ , a map $S: \Im \to \Im$ is referred as a contraction if $\mathfrak{d}(S\mu, S\nu) \leq \alpha\, \mathfrak{d}(\mu, \nu)$ for all $\mu, \nu \in \Im$, for some real number $\alpha$, $0 \leq \alpha < 1$ and is known as contractive if $\mathfrak{d}(S\mu, S\nu) < \mathfrak{d}(\mu, \nu)$ for all $\mu, \nu \in \Im$ with $\mu \neq \nu$. A contraction / contractive mapping is uniformly continuous but not conversely. According to 'Banach contraction principle': "**If $S: \mathfrak{I} \to \mathfrak{I}$ is a contraction mapping on a complete metric space $(\mathfrak{I}, \mathfrak{d})$ then S has a unique fixed point in $\mathfrak{I}$".**

Note that if $S: \Im \to \Im$, where $(\Im, \mathfrak{d})$ is a complete 'metric space', satisfy the inequality $\mathfrak{d}(S\mu, S\nu) < \mathfrak{d}(\mu, \nu)$ for all $\mu, \nu \in \Im$ then $S$ need not have a fixed point. For example, the map $S: [1, \infty) \to [1, \infty)$ such that $S\mu = \mu + \frac{1}{\mu}$ , has no fixed point, although $|S\mu - S\nu| = |\mu - \nu|\left(1 - \frac{1}{\mu\nu}\right) < |\mu - \nu|$ and $[1, \infty)$ is complete. In 1961, Edelstein [15] extended the 'Banach's contraction principle' for 'contractive mapping' and established an important fixed-point theorem for 'contractive mappings' defined on a compact metric space.

'Banach contraction principle' is the most straightforward and useful results in fixed-point theory. This result spurred academicians to attempt its generalization and extension so that its range of applications should be maximized. Since then, several application issues in the mathematics and engineering have been resolved using fixed point theory. This finding has been expanded upon by numerous researchers in variety of 'metric spaces' such as 'dislocated metric space', 'dislocated quasi metric space', 'partial metric space', 'cone metric space', 'b-metric space', 'extended b-metric space' and so forth, by utilizing various contractions and mappings. Many metrical fixed-point theorems exist for mappings that meet specific

contraction type requirements. One analyzes a sequence of iterates in all these theorems, which, because of contraction conditions, turns into a Cauchy sequence with a definite limit as a fixed point of the mapping in a 'complete metric space'. For common fixed-point theorems, a pair of sequences of iterates works well. Some of the generalized 'metric spaces' are defined as follows:

### 1.1. Various Generalized Metric Spaces

**Definition 1.1.1.** *For a non-empty set* $\Im$*, let* $\eth: \Im \times \Im \to [0, \infty)$ *be a mapping fulfilling the below mentioned requirements:*

($\eth$1) $\eth(\mu, \mu) = 0$;

($\eth$2) $\eth(\mu, \nu) = \eth(\nu, \mu) = 0$ *implies* $\mu = \nu$*;*

($\eth$3) $\eth(\mu, \nu) = \eth(\nu, \mu)$ *for all* $\mu, \nu \in \Im$*;*

($\eth$4) $\eth(\mu, \nu) \leq \eth(\mu, \xi) + \eth(\xi, \nu)$ *for all* $\mu, \nu, \xi \in \Im$.

If $\eth$ satisfies conditions from ($\eth$1) to ($\eth$4) then it is referred as a 'metric' on $\Im$ ($Frèchet$ [18]). If it fulfils ($\eth$1), ($\eth$2) and ($\eth$4), it is referred as a 'quasi metric' on $\Im$ (Wilson [42]). If $\eth$ meets the conditions ($\eth$2) to ($\eth$4) then it is referred as a 'dislocated metric' (simply $\eth$-metric*)* on $\Im$ (Hitzler and Seda [19]) and if $\eth$ fulfils only ($\eth$2) and ($\eth$4) then it is called 'dislocated quasi metric' (simply $\eth_q$-metric on $\Im$ (Zeyada et al. [43]).

The writings of Bakhtin [6] and Bourbaki [5] are where the notion of 'b-metric' first appeared. Eventually Czerwik [12] precisely defined 'b-metric space' by providing an axiom that was weaker than the triangular inequality.

**Definition 1.1.2.** *(Bakhtin [6], Czerwik [12]) Given that* $s \geq 1$ *is a real number and* $\Im$ *is a non-empty set, the mapping* $\eth_s: \Im \times \Im \to [0, \infty)$ *is called 'b-metric' if it fulfils the following conditions for all* $\mu, \nu, \xi \in \Im$*:*

(b1) $\eth_s(\mu, \mu) = 0$;

(b2) $\eth_s(\mu, \nu) = \eth_s(\nu, \mu) = 0$ *implies* $\mu = \nu$*;*

(b3) $\eth_s(\mu, \nu) = \eth_s(\nu, \mu)$*;*

(b4) $\eth_s(\mu, \nu) \leq s[\eth_s(\mu, \xi) + \eth_s(\xi, \nu)]$.

The pair $(\Im, \eth_s)$ is referred to as a 'b-metric space'.

- The notion of 'b-metric space' coincides with the notion of usual 'metric space' if $s = 1$. Consequently, all metric spaces are b-metric spaces; however, the converse is not true [25].
- A 'b-metric' $\mathfrak{d}_s$ is not continuous [41], however a metric $\mathfrak{d}$ is known to be continuous.
- If $\mathfrak{d}_s$ satisfies the conditions (b2) and (b4) then it is called 'dislocated quasi b-metric' (simply $\mathfrak{d}_{qs}$-metric, introduced by Rahman and Sarwar [33].

As an extension of 'b-metric space', Kamran et al. [26] recently presented the idea of 'extended b-metric space'.

**Definition 1.1.3**.*(Kamran et al. [26]) Given that* $\Im$ *is a non-empty set and* $\theta: \Im \times \Im \to [1, \infty)$, *the mapping* $\mathfrak{d}_\theta: \Im \times \Im \to [0, \infty)$ *is known as 'extended b-metric' if it fulfils the below mentioned properties for all* $\mu, \nu, \xi \in \Im$ :

$(\mathfrak{d}_\theta 1)$ $\mathfrak{d}_\theta(\mu, \mu) = 0$;

$(\mathfrak{d}_\theta 2)$ $\mathfrak{d}_\theta(\mu, \nu) = \mathfrak{d}_\theta(\nu, \mu) = 0$ *implies* $\mu = \nu$;

$(\mathfrak{d}_\theta 3)$ $\mathfrak{d}_\theta(\mu, \nu) = \mathfrak{d}_\theta(\nu, \mu)$;

$(\mathfrak{d}_\theta 4)$ $\mathfrak{d}_\theta(\mu, \nu) \leq \theta(\mu, \nu)[\mathfrak{d}_\theta(\mu, \xi) + \mathfrak{d}_\theta(\xi, \nu)]$.

The pair $(\Im, \mathfrak{d}_\theta)$ is referred to as an 'extended b-metric space'.

- For $\theta(\mu, \nu) = s \geq 1$, for all $\mu, \nu \in \Im$, the definition of 'extended b-metric space' and 'b-metric' coincide and it reduces to be usual metric if $s = 1$.
- An 'extended b-metric' $\mathfrak{d}_\theta$ is not continuous.

Following are some examples of various generalized metric spaces.

**Example 1.1.1.***(Rahman and Sarwar [32]) Let* $\Im = [0, \infty)$ *and define* $\mathfrak{d}: \Im \times \Im \to [0, \infty)$ *by* $\mathfrak{d}(\mu, \nu) = max\{\mu, \nu\}$ , $for\ all\ \mu, \nu \in \Im$. *Then* $\mathfrak{d}$ *is a 'dislocated metric' on* $\Im$ *but not a 'metric' on* $\Im$.

**Example 1.1.2.***(Rahman and Sarwar [32]) Let* $\Im = [0, 1]$ *and define* $\mathfrak{d}: \Im \times \Im \to [0, \infty)$ *by* $\mathfrak{d}(\mu, \nu) = |\mu|, for\ all\ \mu, \nu \in \Im$. *Then* $\mathfrak{d}$ *is a 'dislocated quasi- metric' on* $\Im$ *but not a 'metric' on* $\Im$.

**Example 1.1.3.***(Kir and Kizitune [25]) Let* $\Im = \{0, 1, 2\}$ *and define* $\mathfrak{d}: \Im \times \Im \to [0, \infty)$ *as* $\mathfrak{d}(0, 0) = \mathfrak{d}(1, 1) = \mathfrak{d}(2, 2) = 0$, $\mathfrak{d}(1, 2) = \mathfrak{d}(2, 1) = \mathfrak{d}(0, 1) = \mathfrak{d}(1, 0) = 1$, $\mathfrak{d}(2, 0) = \mathfrak{d}(0, 2) = \alpha \geq 2$. *Then* $\mathfrak{d}(\mu, \nu) \leq \frac{\alpha}{2}[\mathfrak{d}(\mu, \xi) + \mathfrak{d}(\xi, \nu)], for\ all\ \mu, \nu, \xi \in \Im$. *Note that for* $s = \frac{\alpha}{2}, where\ \alpha \geq 2, (\Im, \mathfrak{d})$ *is a 'b- metric space' but not a metric space for* $\alpha > 2$.

**Example 1.1.4.***(Rahman and Sarwar [33]) Let* $\Im = R$ *and define* $\mathfrak{d}: \Im \times \Im \to [0, \infty)$ *by* $\mathfrak{d}(\mu, \nu) = |2\mu - \nu|^2 + |2\mu + \nu|^2, for\ all\ \mu, \nu \in \Im$. *Then* $\mathfrak{d}$ *is a 'dislocated quasi b- metric' on* $\Im$ *with the coefficient s=2.*

**Example 1.1.5.** *(Kamran et. al [26]) Let* $\Im = [0, \infty)$ *and define* $\mathfrak{d}_\theta : \Im \times \Im \to [0, \infty)$ *and* $\theta: \Im \times \Im \to [1, \infty)$ *by* $\mathfrak{d}_\theta(\mu, \nu) = (\mu - \nu)^2,\ \theta(\mu, \nu) = \mu + \nu + 2,\ for\ all\ \mu, \nu \in \Im$. *Then* $(\Im, \mathfrak{d}_\theta)$ *is an 'extended b- metric space'.*

## 1.2. Basic Concepts and Definitions

The fundamental ideas and terminology utilized throughout the research work are now discussed. Basic topological notions like 'convergence', 'Cauchy sequence' and 'completeness' in 'dislocated/ dislocated quasi metric space' are defined as follows.

**Definition 1.2.1.***(Hitzler and Seda [19]) Let* $\{\xi_j\}$ *be a sequence in 'dislocated metric space'* $(\Im, \mathfrak{d})$ *then*

- $\{\xi_j\}$ $\mathfrak{d}$ *- converges to* $\xi \in \Im$ *if* $\lim_{j\to\infty} \mathfrak{d}(\xi_j, \xi) = 0$. *Here* $'\xi'$ *is called* $\mathfrak{d}$*-limit of* $\{\xi_j\}$ *and we write* $\xi_j \to \xi$.

- $\{\xi_j\}$ *is referred as a 'Cauchy sequence' in* $(\Im, \mathfrak{d})$ *if for given* $\epsilon > 0$ $there\ exists\ n_0 \in N$ *such that* $\mathfrak{d}(\xi_i, \xi_j) < \epsilon, for\ all\ i, j \geq n_0$, *that is,* $\lim_{i,j\to\infty} \mathfrak{d}(\xi_i, \xi_j) = 0$.

- *A 'dislocated metric space'* $(\Im, \mathfrak{d})$ *is known as complete if every 'Cauchy sequence' of points in* $\Im$ *converges to a point in* $\Im$.

- *Every convergent sequence in 'dislocated metric space' has a unique limit.*

**Definition 1.2.2.***(Zeyada et al. [43]) Let* $\{\xi_j\}$ *be a sequence in 'dislocated quasi metric space'* $(\Im, \mathfrak{d})$ *then*

- $\{\xi_j\}$ $\mathfrak{d}_q$*- converges to* $\xi \in \Im$ *if* $\lim_{j\to\infty} \mathfrak{d}(\xi_j, \xi) = \lim_{j\to\infty} \mathfrak{d}(\xi, \xi_j) = 0$. *Here* $'\xi'$ *is called* $\mathfrak{d}_q$*-limit of* $\{\xi_j\}$ *and we write* $\xi_j \to \xi$.

- $\{\xi_j\}$ *is referred as a 'Cauchy sequence' in* $(\Im, \mathfrak{d})$ *if for given* $\epsilon > 0$ $there\ exists\ n_0 \in N$ *such that* $\mathfrak{d}(\xi_i, \xi_j) < \epsilon$ *or* $\mathfrak{d}(\xi_j, \xi_i) < \epsilon, for\ all\ i, j \geq n_0$, *that is,*

  $$\lim_{i,j\to\infty} \mathfrak{d}(\xi_i, \xi_j) = \lim_{i,j\to\infty} \mathfrak{d}(\xi_j, \xi_i) = 0.$$

- *In above definition, if we replace* $\mathfrak{d}(\xi_i, \xi_j) < \epsilon$ *or* $\mathfrak{d}(\xi_j, \xi_i) < \epsilon$ *by* $max\{\mathfrak{d}(\xi_i, \xi_j), \mathfrak{d}(\xi_j, \xi_i)\} < \epsilon, the\ sequence\ \{\xi_j\}$ *in* $\mathfrak{d}_q$*- metric space* $(\Im, \mathfrak{d})$ *is called 'bi' Cauchy.*

- *A* $\mathfrak{d}_q$*-metric space* $(\Im, \mathfrak{d})$ *is known as complete if every 'Cauchy sequence' is a* $\mathfrak{d}_q$*-convergent sequence.*

- *Every convergent sequence in* $\mathfrak{d}_q$*-'metric space' has a unique limit.*

- *Every convergent sequence in a* $\mathfrak{d}_q$*- 'metric space' is 'bi' Cauchy.*

The above definitions can be carried forward to 'b-metric spaces' and 'extended b-metric' spaces.

## 2. Literature Review

The topic of fixed-point theory has seen a great deal of research interest. However, only a handful of fixed-point results under generalized contraction mappings are highlighted by us in various complete generalized 'metric spaces' which are relevant to the present research work. Several significant findings are examined hereunder:

In 1961, Edelstein [15] extended the 'Banach's contraction principle' for 'contractive mapping' on a 'compact metric space' as follows.

**Theorem 2.1**.*(Edelstein [15]) Let* $S: \Im \to \Im$ *be a self-mapping on a compact metric space* $(\Im, \mathfrak{d})$ *satisfying:* $\mathfrak{d}(S\mu, S\nu) < \mathfrak{d}(\mu, \nu)$ for all $\mu, \nu \in \Im$, $\mu \neq \nu$, *then S admits a unique fixed point in* $\Im$. *Also,* $the\ sequence\ \{S^n(\mu)\}$ *converges to the unique fixed point of S for any* $\mu \in \Im$.

In 1968, Kannan [24] introduced a 'contraction map' that admits a unique fixed-point while having discontinuity in its domain in a 'complete metric space'.

**Theorem 2.2**.*(Kannan [24]) Let* $S: \Im \to \Im$ *be a self-mapping on a complete metric space* $(\Im, \mathfrak{d})$ *satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq \alpha[\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\nu, S\nu)] \ldots\ldots\ldots\ldots\ldots (1)$$

for all $\mu, \nu \in \Im$, *where* $0 \leq \alpha < \frac{1}{2}$. *Then S admits a unique fixed point in* $\Im$.

A mapping that satisfies (1) is referred to as 'Kannan type' mapping.

Later Chatterjea, 1972 [13] introduced a similar type of 'contraction map' having discontinuity in its domain but admitting a unique fixed point in a 'complete metric space'.

**Theorem 2.3**.*(Chatterjea [13]) Let* $S: \Im \to \Im$ *be a self-mapping on a complete metric space* $(\Im, \mathfrak{d})$ *satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq \beta[\, \mathfrak{d}(\mu, S\nu) + \mathfrak{d}(\nu, S\mu)] \ldots\ldots\ldots\ldots (2)$$

for all $\mu, \nu \in \Im$, *where* $0 \leq \beta < \frac{1}{2}$. *Then S admits a unique fixed point in* $\Im$.

A 'Chatterjea type' mapping is one that fulfils (2).

Several authors have proposed various contraction type mappings and fixed-point theorems in a 'complete metric space' (Reich [36], Bianchini [10], Hardy and Rogers [21], Ciric [11]).

**Theorem 2.4.***(Reich [36]) Let* $S: \Im \to \Im$ *be a self-mapping on a complete metric space* $(\Im, \mathfrak{d})$ *satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq \alpha\, \mathfrak{d}(\mu, S\mu) + \beta\, \mathfrak{d}(\nu, S\nu) + \gamma \mathfrak{d}(\mu, \nu) \ldots\ldots\ldots\ldots (3)$$

for all $\mu, \nu \in \Im$, *where* $\alpha, \beta, \gamma \geq 0$ $with$ $\alpha + \beta + \gamma < 1$. *Then S admits a unique fixed point in* $\Im$.

A 'Reich type' mapping is a mapping that fulfils (3).

**Theorem 2.5**.*(Bianchini [10]) Let* $S: \Im \to \Im$ *be a self-mapping on a complete metric space* $(\Im, \mathfrak{d})$ *satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq \eta\, max\, \{\, \mathfrak{d}(\mu, S\mu), \mathfrak{d}(\nu, S\nu)\} \ldots\ldots\ldots\ldots (4)$$

for all $\mu, \nu \in \Im$, *where* $0 \leq \eta < 1$. *Then S admits a unique fixed point in* $\Im$.

A mapping is called 'Bianchini type' if it fulfils equation (4).

**Theorem 2.6**.*(Hardy and Rogers [21]) Let* $S: \Im \to \Im$ *be a self-mapping on a complete metric space* $(\Im, \mathfrak{d})$ *satisfying:*

$\mathfrak{d}(S\mu, S\nu) \leq \eta_1\, \mathfrak{d}(\mu, S\mu) + \eta_2\, \mathfrak{d}(\nu, S\nu) + \eta_3 \mathfrak{d}(\mu, S\nu) + \eta_4 \mathfrak{d}(\nu, S\mu) + \eta_5 \mathfrak{d}(\mu, \nu) \ldots\ldots\ldots (5)$

for all $\mu, \nu \in \Im$, *where* $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5 \geq 0$ *and* $\eta = \eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 < 1$. *Then S has a unique fixed point in* $\Im$.

A mapping that satisfies (5) is referred to as 'Hardy and Rogers type' mapping.

**Definition 2.1.** *(Ciric [11]) Let* $(\Im, \mathfrak{d})$ *be a metric space, a self-mapping* $S: \Im \to \Im$ *is known as a 'generalized contraction' if and only if for every* $\mu, \nu \in \Im$, *there exist* $\eta_1, \eta_2, \eta_3, \eta_4$ *such that sup* $\{\eta_1 + \eta_2 + \eta_3 + 2\eta_4 : \mu, \nu \in \Im\} < 1$ *and*

$$\mathfrak{d}(S\mu, Sv) \leq \eta_1 \mathfrak{d}(\mu, v) + \eta_2\, \mathfrak{d}(\mu, S\mu) + \eta_3 \mathfrak{d}(v, Sv) + \eta_4[\mathfrak{d}(\mu, Sv) + \mathfrak{d}(v, S\mu)]. \ldots\ldots\ldots (6)$$

A unique fixed-point theorem for a mapping that meets condition (6) in metric spaces was proved by Ciric [11].

In 1975, Dass and Gupta [14] demonstrated a generalization of 'Banach contraction principle' employing rational expression in a 'complete metric space'.

**Theorem 2.7.***(Dass and Gupta [14]) Let $S: \Im \to \Im$ be a continuous self-mapping on a complete metric space $(\Im, \mathfrak{d})$. If there exist $p, q \in [0,1)$ with $p + q < 1$ such that*

$$\mathfrak{d}(S\mu, Sv) \leq p\, \mathfrak{d}(v, Sv) \frac{[1+\mathfrak{d}(\mu, S\mu)]}{1+\mathfrak{d}(\mu, v)} + q\, \mathfrak{d}(\mu, v),$$

for all $\mu, v \in \Im$ *, then S admits a unique fixed point in* $\Im$*. Also, the sequence* $\{S^n\mu\}$ *converges to the fixed point* $\mu_0$ *for all* $\mu \in \Im$.

Another useful generalization of 'Banach contraction principle' using rational expression was given by Jaggi, 1976) [23].

**Theorem 2.8**.*(Jaggi [23]) Let $S: \Im \to \Im$ be a continuous self-mapping on a complete metric space $(\Im, \mathfrak{d})$. If there exist $p, q \in [0,1)$ with $p + q < 1$ such that*

$$\mathfrak{d}(S\mu, Sv) \leq p\, \mathfrak{d}(\mu, S\mu) \frac{\mathfrak{d}(v, Sv)}{\mathfrak{d}(\mu, v)} + q\, \mathfrak{d}(\mu, v),$$

for all $\mu, v \in \Im$ *with* $\mu \neq v$*, then S admits a unique fixed point in* $\Im$.

Following result was given by Fisher, 1978 [17].

**Theorem 2.9**.*(Fisher [17]) If $S: \Im \to \Im$ is a continuous self-mapping on a compact metric space $(\Im, \mathfrak{d})$ satisfying:*

$$\mathfrak{d}(S\mu, Sv) < \frac{1}{2}[\mathfrak{d}(v, S\mu) + \mathfrak{d}(\mu, Sv)] \text{ for all } \mu, v \in \Im,\ \mu \neq v,$$

*then S has a unique fixed point in* $\Im$.

In 1977, Rhodes [37] in his paper, conducted a thorough analysis and comparison of various contraction / contractive conditions that are dispersed across the literature.

In 1993, Czerwik [12] defined ‘b-metric space’ and established ‘Banach’s contraction theorem’ in this space. Following Czerwik many authors proved numerous fixed-point results in ‘b-metric space’.

In 2000, Hitzler and Seda [19] first described dislocated metric spaces, who also generalized the ‘Banach contraction principle’ in this space. The notion of ‘dislocated quasi metric space’ was presented by Zeyada et al., 2005 [43], who also expanded on Hitzler and Seda’s findings in this context and proved that in a ‘complete dislocated quasi metric space’, every continuous contraction $S: \Im \to \Im$ admits a unique fixed-point. In this space, Aage and Salunke, 2008) [1] demonstrated fixed point results for generalized contraction mapping and Kannan mapping. Following common fixed-point theorem was also proved.

**Theorem 2.10**.*(Aage and Salunke [1]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated metric space. Let* $S, T: \Im \to \Im$ *be continuous self-mappings satisfying:*

$$\mathfrak{d}(S\mu, Tv) \leq h\, max\{\mathfrak{d}(\mu, v), \mathfrak{d}(\mu, S\mu), \mathfrak{d}(v, Tv)\},$$

*for all* $\mu, v \in \Im$ *and* $0 < h < 1$. *Then S and T have common fixed-point.*

In 2010, Kohli et al. [28] presented the subsequent theorem in ‘complete dislocated quasi metric space’ which generalizes the result of Dass and Gupta, 1975 [14].

**Theorem 2.11**.*(Kohli et al. [28]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated quasi metric space. Let* $S: \Im \to \Im$ *be a continuous self-mapping satisfying:*

$$\mathfrak{d}(S\mu, Sv) \leq p\, \mathfrak{d}(v, Sv) \frac{[1+\mathfrak{d}(\mu, S\mu)]}{1+\mathfrak{d}(\mu, v)} + q\, \mathfrak{d}(\mu, v) + r\, \mathfrak{d}(v, Sv),$$

*for all* $\mu, v \in \Im$ *and where* $p, q, r > 0$ *with* $p + q + r < 1$. *Then S admits a unique fixed-point.*

In 2012, Shrivastava et al. [38] proved fixed-point theorem for Jaggi mapping and the following theorem which generalizes the results of Kannan [24], Chatterjea [13] and Jaggi [23].

**Theorem 2.12**.*(Shrivastava et al. [38]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated quasi metric space. Let* $S: \Im \to \Im$ *be a continuous self-mapping satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq p\,\mathfrak{d}(\mu,\nu) + q\,\mathfrak{d}(\mu, S\mu)\frac{\mathfrak{d}(\nu, S\nu)}{\mathfrak{d}(\mu,\nu)} + r[\,\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\nu, S\nu)] + s[\,\mathfrak{d}(\mu, S\nu) + \mathfrak{d}(\nu, S\mu)]$$

*for all* $\mu, \nu \in \Im$ *with* $\mu \neq \nu$, $p, q, r, s \in [0,1)$ *and* $0 \leq p + q + 2r + 2s < 1$. *Then S has a unique fixed-point.*

The above result was further modified and generalized by Zoto et al., 2012 [44] by adding the factor $t[\,\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\mu,\nu)]$ in the above contraction condition with $0 \leq p + q + 2r + 2s + 2t < 1$. In 2014, Rahman and Sarwar [32] investigated fixed-point results in 'complete dislocated quasi metric space' for different type of contraction condition which unified some existing results in literature for single and a pair of continuous self-mapping.

**Theorem 2.13**.*(Rahman and Sarwar [32]) Let* $S: \Im \to \Im$ *be a continuous self-mapping on a complete dislocated quasi metric space satisfying:*

$$\mathfrak{d}(S\mu, S\nu) \leq \eta_1\mathfrak{d}(\mu,\nu) + \eta_2\mathfrak{d}(\mu, S\nu) + \eta_3\mathfrak{d}(\nu, S\mu) + \eta_4\mathfrak{d}(\nu, S\nu) + \eta_5\mathfrak{d}(\nu, S\nu)\frac{[1 + \mathfrak{d}(\mu, S\mu)]}{1 + \mathfrak{d}(\mu,\nu)}$$
$$+\eta_6\,\frac{\mathfrak{d}(\nu,\ S\nu) + \mathfrak{d}(\nu, S\mu)}{1 + \mathfrak{d}(\nu, S\nu)\mathfrak{d}(\nu, S\mu)} + \eta_7\,\frac{\mathfrak{d}(\mu, S\mu)[1 + \mathfrak{d}(\nu, S\mu)]}{1 + \mathfrak{d}(\mu,\nu) + \mathfrak{d}(\nu, S\nu)}$$

*for all* $\mu, \nu \in \Im$, *where* $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7 \geq 0$ *with* $\eta_1 + 2(\eta_2 + \eta_3) + \eta_4 + \eta_5 + 3\eta_6 + \eta_7 < 1$. *Then S admits a unique fixed-point.*

Rahman and Sarwar, 2016 [33] introduced the notion of 'dislocated quasi b-metric space' as a 'b-metric space' generalization and established 'Banach's contraction principle' and other well-known fixed-point theorems for Kannan, Chatterjea type mappings.

In 2017, Kamran et al. [26] motivated by the idea of 'b-metric space', introduced 'extended b-metric space' and validated analog of 'Banach contraction principle' under the framework of 'extended b-metric space'.

**Theorem 2.14.** *(Kamran et al. [26]) Let* $S: \Im \to \Im$ *be a mapping, where* $(\Im, \mathfrak{d}_\theta)$ *is a complete extended b-metric space. If there exists* $k \in [0,1)$ *such that* $\mathfrak{d}_\theta(S\mu, S\nu) \leq k\,\mathfrak{d}_\theta(\mu,\nu)$, *for all* $\mu, \nu \in \Im$, *where for each* $\mu_0 \in \Im$, $\lim_{m,n\to\infty} \theta(\mu_n,\ \mu_m) < \frac{1}{k}$, *where* $\mu_n = S^n\mu_0$, $n \in N$. *Then S admits one fixed point p in* $\Im$. *Furthermore, for each* $\nu \in \Im, S^n\nu \to p$.

In 2019, Alqahtani [3] established fixed point theorems of Dass and Gupta [14] and Jaggi [23] in the context of 'extended b-metric space'.

**In (2021), Rahman and Mussa [34]** under the framewok of 'dislocated quasi metric space', proposed common fixed-point result for a pair of continuous self-mappings which extends the findings of Banach [4], Kannan [24], Chatterjea [13], Reich [36], Hardy and Roger [21] and Dass and Gupta [14].

**Theorem 2.15**.*(Rahman and Mussa [34]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated quasi metric space. Let* $S, T : \Im \to \Im$ *be two continuous self-mapping satisfying the condition:*

$$\mathfrak{d}(S\mu, T\nu) \leq \eta_1 \mathfrak{d}(\mu, \nu) + \eta_2[\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\nu, T\nu)] + \eta_3[\mathfrak{d}(\nu, S\mu) + \mathfrak{d}(\mu, T\nu)] + \eta_4 \frac{\mathfrak{d}(\nu, T\nu)[1 + \mathfrak{d}(\mu, S\mu)]}{1 + \mathfrak{d}(\mu, \nu)} + \eta_5 \mathfrak{d}(\nu, S\mu) \frac{[1 + \mathfrak{d}(\mu, T\nu)]}{1 + \mathfrak{d}(\mu, S\nu)} + \eta_6 \mathfrak{d}(\nu, S\mu)$$

*for all* $\mu, \nu \in \Im$, *where* $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6 \geq 0$ *with* $\eta_1 + 2(\eta_2 + \eta_3) + \eta_4 + \eta_5 + \eta_6 < 1$. *Then S and T admit a unique common fixed-point in* $\Im$.

Utilizing a new contraction map without requiring continuity, Mhanna et al., 2022 [31] established new generalization of the fixed-point theorem under the framework of 'dislocated quasi metric' spaces.

**Theorem 2.16**.*(Mhanna et al. [31]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated quasi metric space. Let* $S$: $\Im \to \Im$ *be a self-mapping satisfying the condition:*

$$\mathfrak{d}(S\mu, S\nu) \leq \alpha \, max \begin{cases} 2\,\mathfrak{d}(\mu, \nu), \dfrac{2\,\mathfrak{d}(\mu, S\mu)\mathfrak{d}(\nu, S\nu)}{\mathfrak{d}(\mu, \nu)}, [\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\nu, S\nu)], \\ \left[\dfrac{\mathfrak{d}(\mu, S\nu) + \mathfrak{d}(\nu, S\mu)}{2}\right], [\mathfrak{d}(\mu, S\mu) + \mathfrak{d}(\mu, \nu)], \\ [\mathfrak{d}(\nu, S\nu) + \mathfrak{d}(\mu, \nu), 2[\dfrac{\mathfrak{d}(\mu, S\nu) + \mathfrak{d}(\mu, \nu)}{3}] \end{cases}$$

*for all* $\mu, \nu \in \Im$ *with* $\mathfrak{d}(\mu, \nu) \neq 0$, *and* $0 \leq \alpha < \frac{1}{2}$. *Then S admits a unique fixed point in* $\Im$.

In 2023, Rahman et al. [35] demonstrated a common fixed-point theorem in the setting of 'dislocated quasi b-metric space' for new contraction condition using rational expression. This result enhances and generalizes previous fixed-point results by Isufati [22] and Rahman and Sarwar [33].

**Theorem 2.17**.*(Rahman et al. [35]) Let* $(\Im, \mathfrak{d})$ *be a complete dislocated quasi b-metric space with coefficient* $s \geq 1$. *Let* $S, T : \Im \to \Im$ *be two continuous self-mapping fulfilling the below mentioned condition:*

$$\mathfrak{d}(S\mu, T\nu) \leq c_1 \mathfrak{d}(\mu, \nu) + c_2 \frac{\mathfrak{d}(\nu, T\nu)[1 + \mathfrak{d}(\mu, S\mu)]}{1 + \mathfrak{d}(\mu, \nu)} + c_3 \frac{\mathfrak{d}(\mu, T\nu)\mathfrak{d}(\nu, T\nu)}{s[\mathfrak{d}(\mu, \nu) + \mathfrak{d}(\nu, T\nu)]} +$$

$$c_4 \frac{\mathfrak{d}(\mu, S\mu)\mathfrak{d}(\nu, T\nu)}{1 + \mathfrak{d}(\mu, \nu)} + c_5 \mathfrak{d}(\mu, S\mu) \frac{\mathfrak{d}(\mu, T\nu)}{1 + \mathfrak{d}(\mu, \nu)}$$

*for all* $u, v \Im, where\ c_1, c_2, c_3, c_4, c_5 \geq 0\ with\ sc_1 + c_2 + c_3 + c_4 + s(1 + s)c_5 < 1$. *Then* $S$ *and* $T$ *have a unique common fixed-point in* $\Im$.

## 3. Description of Broad Area

One of the main topics of nonlinear functional analysis is fixed point theory, which has some interesting implications to other areas of mathematics for demonstrating the existence and uniqueness results for certain nonlinear issues that develop across a range of biological and physical systems. The development and refinement of criteria for demonstrating the existence and uniqueness of solutions to various kinds of problems is the main emphasis of pure mathematics; on the other hand, applied mathematics studies the procedures or approaches used to identify these answers. The evolutionary method of demonstrating the existence of a solution to an equation is as simple as framing it as a fixed-point problem. Metric space analysis is among the most effective and potent tools for applying fixed point theorems. The majority of scholars working in the fields of topology and nonlinear analysis are exploring a wide spectrum. The permissible condition for the existence of fixed points depends on the following factors:

- Restriction of domain
- Function definition technique
- Topological characteristics
- Iteration scheme used

The review finds applications in many important fields, including engineering, mathematical economics, operation research, differential equations, and integral equations, which makes the study of metric fixed-point theory important.

## 4. Objectives of the Study

- To establish fixed-point results for various types of contraction conditions using linear and rational expression in ‘dislocated / dislocated quasi metric space’.
- To prove coupled fixed-point results for maps that fulfils contraction condition in ‘dislocated quasi b- metric space’.
- To introduce a new generalized ‘metric space’ and then to establish well-known fixed-point results in this space.
- To study and develop conditions for ‘Banach contraction principle’, Kannan, Chatterjea, Reich, Bianchini and Hardy & Rogers types fixed-point results in different generalized ‘metric spaces’.
- To formulate some generalized fixed-point theorems for single and pair of continuous self-mappings under different contractive conditions using linear and rational expression in compact metric spaces.

## 5. Methodology to be Adopted

The following stages would compose the research approach to be used:

- Stage 1: Review of literature.

- Stage 2: Deciding the generalized ‘metric spaces’ for establishing new fixed-point results.

- Stage 3: Defining new contraction conditions using linear and rational expression for single as well as pair of mappings in chosen space.

- Stage 4: Examining and assigning conditions on the defined mapping that guarantees the existence and uniqueness of fixed-point in the underlying space framework.

- Stage 5: Comparing the existing fixed-point results.

- Stage 6: Introducing a new generalized metric space and developing conditions for well-known contractions in this space.

**6. Expected Outcomes of the Research**

While pursuing PhD program, our main objective is to construct some new contraction conditions using linear and rational expression in various spaces namely: ‘metric space’, ‘dislocated metric space’, ‘dislocated quasi metric space’, ‘dislocated quasi b-metric space’ and ‘extended b-metric space’ and to introduce a new generalized ‘metric space’. The broader the class of functions or spaces that these generalizations cover, the more useful they become and the more application domains they may be used in.
The established results will broaden and expand upon several fixed-point results found in the literature. Future existential issues arising from differential equations, integral equations, systems of linear equations, and numerous other mathematical constructs may find a solution in our findings.

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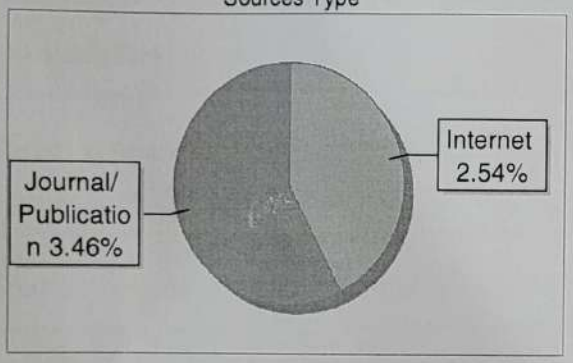


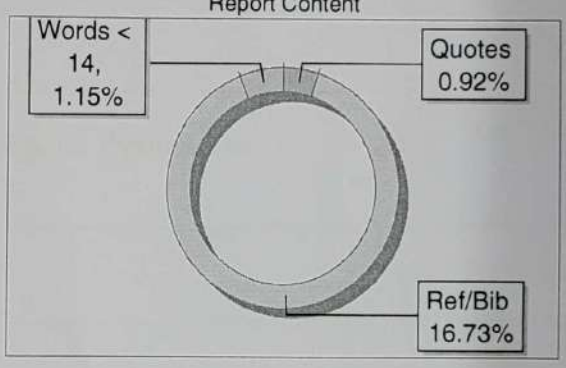


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Annexure-4

# FIXED-POINT THEOREMS IN GENERALIZED METRIC SPACES UNDER DIFFERENT CONTRACTION CONDITIONS

## SHORT SYNOPSIS

*by*

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## ABSTRACT

This short synopsis gives an overview of some fixed-point results in generalized metric spaces under different contraction conditions. Our current research endeavors aim to expand the

Banach contraction principle in many ways for example by easing the conditions or generalizing the underlying metric space and / or relaxing the contraction types. The short synopsis comprises the following **seven sections**.

**Section 1** gives a brief introduction of the history and development of fixed-point theory and is divided into **two subsections**. The definitions of the several generalized metric spaces that have been discussed and introduced in the literature are provided with examples in **Section 1.1**. **Section 1.2** discusses basic concepts and topological notions used in generalized metric spaces.

**Section 2** titled Literature Review summarizes the main work done by various authors related to my present research area.

**Section 3** gives the description of broad area.

**Section 4** addresses various objectives and goals of the research.

**Section 5's** objective is to analyze the research methods.

**Section 6 and 7** discusses expected outcomes of the research and list of references respectively.

**Key words**: Fixed point, Contraction map, Dislocated metric space, Dislocated quasi metric space, b-metric space, Extended b-metric space, Cauchy sequence.



## 1. Introduction

One of the most well-known and progressive areas of nonlinear functional analysis study is the theory of fixed points. It deals with existence and uniqueness of fixed point ($S(\mu) = \mu$), that is, a point that does not change under a given map. A mapping may or may not have any fixed points; if it does, there may be one single fixed point or several. Many types of fixed points such as coincidence point, common fixed point, coupled fixed point and many more, may be found in literature. Fixed point theorems are those that address the presence and characteristics of fixed points. In this field, the first fixed point theorem was established in 1912 by Brouwer

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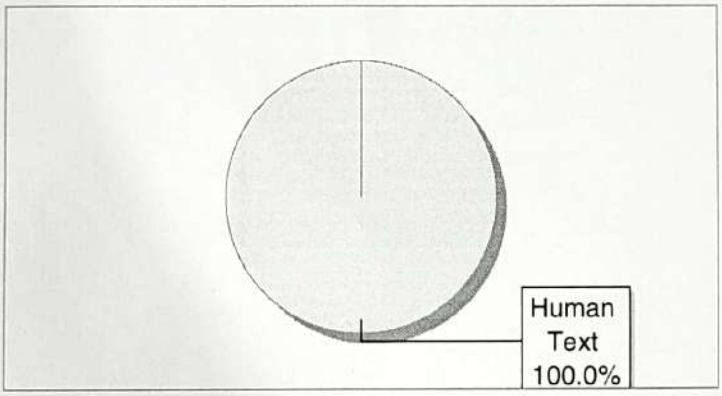


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